# Light-Chronometric Convergence Cosmology: Finite Light-Track Age from a State-Dependent Effective Causal Speed

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We introduce a phenomenological framework in which the cosmic "age" is measured by an accumulated causal track length, $\tau(t) = \int_0^t c_{\rm eff}(t')\,dt'$, where $c_{\rm eff}$ is an effective propagation speed for the matter–photon sector that depends on cosmological state variables (thermodynamic temperature $T$ (or the dimensionless $\Theta = k_{\rm B}T/(h\nu_{\rm H})$ referenced to hydrogen 21 cm) and a geometric phase $\theta$). We fix "convergence" by a single geometric criterion: the finite-future event-horizon radius $R_e(t) = a(t)\int_t^{t_*} c_{\rm eff}(t')/a(t')\,dt'$ tends to zero as $t \to t_*^-$. As a minimal closure we choose $c_{\rm eff}(t) = c_0(1 - t/t_*)^p$ with $p = 2/3$, yielding a terminal light-chronometric age $\tau_\infty = 0.6\,c_0 t_*$ (e.g. $\tau_\infty \simeq 60$ trillion light-years for $t_* \simeq 100$ trillion years). We derive a direct observational reconstruction in spatially flat FRW: $c_{\rm eff}(z) = H(z)\,dD_M/dz$, providing a falsifiable null test using $H(z)$ (cosmic chronometers) and distances (SN Ia/BAO). We discuss consistency requirements from dimensionless-constant bounds and multi-messenger constraints on the gravitational-wave speed, and we treat $T \to 0$ as a zero-thermal-noise limit rather than an attained endpoint.

KEYWORDS: cosmology; varying speed of light; distance–redshift; redshift drift; horizon

## 1. Introduction

The standard "age of the Universe" is defined as a *proper time* since a hot early phase, inferred within a cosmological model. In this work we explore an alternative, operationally motivated clock: the *accumulated causal track length* of light (or any matter–photon signal), measured in units of distance. The motivation is pragmatic: a distance-like measure is (i) directly tied to signal propagation, and (ii) does not depend on conventions such as the Earth year.

We introduce a *light-chronometric age* $\tau(t)$ defined by an effective propagation speed $c_{\rm eff}(t)$,

$$\tau(t) \;\equiv\; \int_0^t c_{\rm eff}(t')\,dt', \qquad (1)$$

and we consider cosmologies in which $\tau(t)$ approaches a *finite* terminal value $\tau_\infty$ at a finite (or effectively finite) future time $t_*$. We call this class *light-chronometric convergence cosmology.*

A central "framework declaration" (made explicit in appendix A) is that we do *not* vary the fundamental constant $c_0$ by fiat. Rather, we treat $c_{\rm eff}$ as an effective causal speed for the matter–photon sector (e.g. in a disformal/bimetric description), consistent with the broader literature on varying-speed-of-light (VSL) and varying-constant frameworks.[1–3,5,10] We also keep in mind the conceptual debate on dimensional vs. dimensionless "constants".[10,11]

To connect with the "60 trillion light-years" target value, we will present a minimal toy model in which

$$\tau_\infty \;\simeq\; 60\ {\rm Tly}, \qquad (2)$$

where ${\rm Tly} \equiv 10^{12}$ ly. This choice is not an observational claim; it is a convenient normalization for a phenomenological closure.

**Contributions.** The main contributions of this paper are summarized as follows:

(i) **Definition:** we define the light-chronometric age $\tau(t) = \int_0^t c_{\rm eff}(t')\,dt'$ as a distance-like measure of cosmic age.

(ii) **Convergence criterion:** we fix convergence by the single geometric condition $R_e(t) \to 0$ at a finite future time.

(iii) **Observational tests:** we derive a direct reconstruction null test, $c_{\rm eff}(z) = H(z)\,dD_M/dz$, together with a redshift-drift relation that can separate changes in $c_{\rm eff}$ from changes in the background expansion law.

## 2. State-dependent effective causal speed

### 2.1 From fixed $c_0$ to an effective $c_{\rm eff}(T,\theta)$

We model the effective causal speed as a function of cosmological state variables, schematically

$$c_{\rm eff} \;=\; c_0\, f(T,\theta), \qquad 0 < f \le 1, \qquad (3)$$

where $T$ is a coarse-grained cosmic temperature (for the relevant sector) and $\theta$ is a dimensionless "geometric phase" parameter (for example, the phase

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of a background scalar field or an order parameter controlling a disformal factor). Equation (3) is intended as an effective description. Concrete implementations exist in bimetric or disformal frameworks where matter couples to a "matter metric" $\hat{g}_{\mu\nu}$ related to the gravitational metric $g_{\mu\nu}$.[4,5)]

*2.2 Light-chronometric closure at finite $\tau_\infty$*

Consider a future time $t_*$ at which $c_{\rm eff}$ tends to zero sufficiently fast that $\tau(t)$ converges:

$$\tau_\infty \equiv \lim_{t\to t_*^-} \tau(t) < \infty. \tag{4}$$

A simple one-parameter ansatz is

$$c_{\rm eff}(t) = c_0\left(1-\frac{t}{t_*}\right)^p, \qquad p>0, \tag{5}$$

which yields

$$\tau(t) = \frac{c_0 t_*}{p+1}\left[1-\left(1-\frac{t}{t_*}\right)^{p+1}\right], \qquad \tau_\infty = \frac{c_0 t_*}{p+1}. \tag{6}$$

Imposing the "60 Tly at 100 Tyr" normalization,

$$\tau_\infty = 0.6\,c_0 t_* \quad\Longrightarrow\quad p=\frac{2}{3}. \tag{7}$$

With this choice, the approach to convergence is

$$\tau_\infty - \tau(t) = \frac{3}{5}c_0 t_*\left(1-\frac{t}{t_*}\right)^{5/3}. \tag{8}$$

A "threshold" such as $c_{\rm eff} \approx 60\ {\rm km\,h^{-1}}$ can be translated into a remaining light-track

$$\frac{\tau_\infty-\tau}{\tau_\infty} = \left(\frac{c_{\rm eff}}{c_0}\right)^{\frac{p+1}{p}} \overset{p=2/3}{=} \left(\frac{c_{\rm eff}}{c_0}\right)^{5/2}. \tag{9}$$

For $c_{\rm eff}/c_0 \sim 5.6\times10^{-8}$ (corresponding to $60\ {\rm km\,h^{-1}}$), this fraction is $\sim 10^{-18}$, i.e. the remaining light-track is astronomically small compared to $\tau_\infty$. This illustrates how a dramatic late-time slowdown can occur while leaving almost all of $\tau_\infty$ already accumulated.

*2.3 A conservative delayed-decay closure (optional)*

The one-parameter power law (5) is a minimal toy closure. However, if $c_{\rm eff}$ is interpreted as governing long-distance photon/matter signal propagation, current distance data may already constrain sizable deviations from $c_0$ at late times. A conservative variant that preserves $c_{\rm eff} \simeq c_0$ throughout the early and present universe while still achieving a finite $\tau_\infty$ is a *delayed* decay model:

$$c_{\rm eff}(t) = \begin{cases} c_0, & 0\le t\le t_c,\\ c_0\left(1-\dfrac{t-t_c}{t_*-t_c}\right)^p, & t_c<t<t_*,\\ 0, & t\ge t_*,\end{cases} \tag{10}$$

with $p>0$ and a transition time $t_c$. The terminal light-track is then

$$\tau_\infty = c_0 t_c + \frac{c_0(t_*-t_c)}{p+1}. \tag{11}$$

Imposing $\tau_\infty = 0.6\,c_0 t_*$ yields a simple relation

$$\frac{t_c}{t_*} = 0.6 - \frac{0.4}{p}, \qquad p\ge\frac{2}{3}. \tag{12}$$

For example, $p=4$ implies $t_c = 0.5\,t_*$, i.e. $c_{\rm eff}\approx c_0$ for half the lifetime and a rapid decay thereafter. The original one-parameter model (5) corresponds to the special case $t_c=0$.

**Order-of-magnitude consistency with present distance data.** All currently observed photons were emitted at times $t_{\rm em}\le t_0$, where $t_0$ is the present cosmic age (e.g. $t_0 \simeq 13.8\,{\rm Gyr}$[18)]). For the piecewise delayed model (10), if the decay begins strictly in the future, $t_c>t_0$, then $c_{\rm eff}=c_0$ along the entire light path from $t_{\rm em}$ to $t_0$ and standard distance measures are recovered exactly. Hence $t_c>t_0$ is a sufficient conservative condition to avoid conflicts with existing distance–redshift data. Using (12), this condition becomes

$$p > \frac{0.4}{0.6 - t_0/t_*}, \tag{13}$$

which for $t_* = 100\,{\rm Tyr}$ and $t_0\simeq 13.8\,{\rm Gyr}$ gives $p \gtrsim 0.6668$. More generally, for smooth deviations one expects a fractional distance change bounded by the typical fractional deviation of $c_{\rm eff}$ over the probed redshift range; percent-level distance data therefore motivate keeping $|c_{\rm eff}/c_0 - 1| \ll 10^{-2}$ at $z \lesssim \mathcal{O}(1)$ (see e.g. SN Ia and BAO distance compilations[16,17)]).

*2.3.1 Smooth delayed decay and a percent-level distance bound*

The piecewise model (10) enforces $c_{\rm eff}=c_0$ exactly before $t_c$. To allow a controlled "leakage" (e.g. from a finite-width transition in an underlying field theory) we adopt a smooth broken power-law parameterization in terms of the scale factor $a=1/(1+z)$:

$$\frac{c_{\rm eff}(a)}{c_0} \equiv f(a) = \left[1+\left(\frac{a}{a_c}\right)^n\right]^{-p/n}, \tag{14}$$

where $a_c$ is a transition scale factor (corresponding to $z_c = a_c^{-1}-1$), $n$ controls the sharpness of the

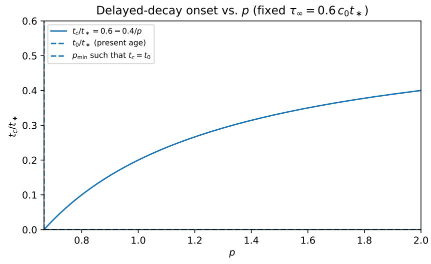


Fig. 1. Transition time $t_c/t_*$ implied by the delayed-decay closure (12) (fixed $\tau_\infty = 0.6\,c_0 t_*$) as a function of $p$. The horizontal line marks the present age $t_0/t_*$ for $t_0 \simeq 13.8\,$Gyr and $t_* = 100\,$Tyr. The region $t_c > t_0$ ensures no modification to current distance measurements in the piecewise model (10).

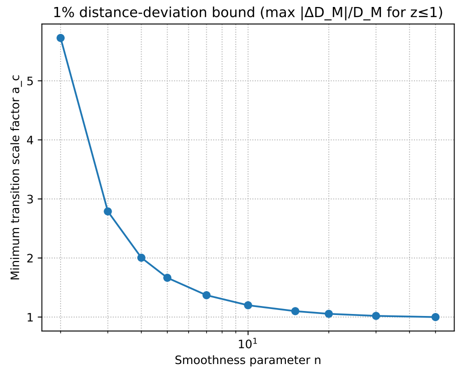


Fig. 2. Percent-level distance-consistency bound for the smooth delayed model (14) with $p = 2/3$. The curve shows the minimum transition scale factor $a_c$ required to keep the maximum fractional change in the flat ΛCDM transverse comoving distance $D_M(z)$ below 1% for $0 < z \leq 1$. Larger $a_c$ (later onset, i.e. more negative $z_c$) or larger $n$ (sharper transition) suppresses "leakage" of the future slowdown into the observed universe.

transition, and $p$ sets the asymptotic decay $c_{\rm eff} \propto a^{-p}$ for $a \gg a_c$. In the limit $n \to \infty$ the function approaches the sharp delayed-decay form, $f(a) \to 1$ for $a < a_c$ and $f(a) \to (a/a_c)^{-p}$ for $a > a_c$.

Since standard late-time cosmologies approach an asymptotically de Sitter expansion, $a \propto e^{H_\Lambda t}$, an asymptotic decay $c_{\rm eff} \propto a^{-p}$ implies $\tau_\infty = \int^\infty c_{\rm eff}(t)\,dt$ converges for $p > 0$.

A convenient "no-impact" criterion on current distance data is

$$\max_{0<z\leq 1} \left| \frac{D_M^{\rm smooth}(z) - D_M^{(0)}(z)}{D_M^{(0)}(z)} \right| < 10^{-2}, \tag{15}$$

where $D_M^{(0)}$ denotes the standard ($c_{\rm eff} = c_0$) prediction in a fiducial flat ΛCDM background. Figure 2 shows the corresponding boundary in the $(n, a_c)$ plane for $p = 2/3$. For example, $n = 10$ requires $a_c \gtrsim 1.2$ ($z_c \lesssim -0.17$), while $n = 5$ requires $a_c \gtrsim 1.7$.

Within the decay phase ($t > t_c$), the remaining light-track to $t_*$ obeys the same scaling as (9) but with $t \mapsto t - t_c$ and $t_* \mapsto t_* - t_c$.

## 3. Causal convergence criterion

### 3.1 *Fixing the convergence definition:* $R_e(t) \to 0$

To define "convergence" in a way that is both geometric and testable, we adopt a single criterion: the future event-horizon radius tends to zero,

$$R_e(t) \to 0 \quad \text{as} \quad t \to t_*^-. \tag{16}$$

For a spatially homogeneous FRW background with scale factor $a(t)$, we define a *finite-future* event horizon by truncating the upper limit at $t_*$,

$$R_e(t) = a(t) \int_t^{t_*} \frac{c_{\rm eff}(t')}{a(t')}\,dt'. \tag{17}$$

If $a(t)$ remains finite and nonzero near $t_*$ and if $c_{\rm eff}(t)$ is integrable on $[t, t_*]$, then $R_e(t) \to 0$ follows. For the power-law model (5) with any $p > 0$, the integral behaves as $(t_* - t)^{p+1}$, so (16) holds.

Equation (16) does *not* assert that the global spatial volume shrinks to a point; rather it asserts that the *causal domain* of any comoving observer collapses to zero size within the effective matter–photon lightcone. In other words, causal communication "freezes out".

### 3.2 *The* $T \to 0^+$ *limit*

In the motivating narrative, the universe cools toward absolute zero.

In a standard adiabatically expanding FRW universe, the radiation temperature scales as $T \propto a^{-1}$ (equivalently $\dot{T} = -HT$), so $T = 0$ is approached only as an asymptotic limit.

In the SI, the kelvin is defined by fixing the numerical value of the Boltzmann constant $k_{\rm B}$; thus $k_{\rm B}T$ is an energy scale independent of any particular substance (water, helium, etc.)[26] For inter-civilizational clarity we further introduce a dimensionless temperature referenced to the hydrogen hyperfine transition (21-cm line), a standard universal marker used in interstellar messaging:[27,28]

$$\Theta \equiv \frac{k_{\rm B}T}{h\nu_{\rm H}}, \tag{18}$$

where $\nu_{\rm H}$ is the 1.42 GHz spin-flip frequency of neutral hydrogen. In this notation the convergence boundary is equivalently expressed as $\Theta \to 0^+$.

Thermodynamically, the third law (unattainability principle) implies that reaching $T = 0$ in finite time is generically impossible; $T = 0$ is best treated as a limit $T \to 0^+$.[22] Accordingly, we interpret $c_{\rm eff}(T,\theta)$ approaching zero as an asymptotic, zero-thermal-noise limit in which thermal fluctuations are suppressed, but quantum vacuum structure does not literally "disappear."

## 4. Effective energy scale and gravitational coupling

### 4.1 *Dynamic $E = mc^2$ with $c_{\rm eff}$*

A frequently cited intuition is that $E = mc^2$ becomes "dynamic" when the causal speed becomes state dependent. Within our framework this is expressed as an *effective rest-energy scale*

$$E_\tau(t) \;\equiv\; m\,c_{\rm eff}^2(t) = mc_0^2 f^2(T,\theta), \qquad (19)$$

while the fundamental mass parameter $m$ and the fundamental constant $c_0$ remain fixed. A time dependence in $E_\tau$ implies an energy exchange with the sector controlling $f(T,\theta)$. In a consistent action-based model this exchange is encoded in the coupled field equations and Bianchi identities.[5,10]

### 4.2 *Strong-coupling indicator from $c_{\rm eff}$ in the field equations*

The Einstein field equation is

$$G_{\mu\nu} = \frac{8\pi G}{c_0^4} T_{\mu\nu}. \qquad (20)$$

If one naively substitutes $c_0 \to c_{\rm eff}(t)$ in the prefactor, one obtains an *effective* coupling

$$\kappa_{\rm eff}(t) \equiv \frac{8\pi G}{c_{\rm eff}^4(t)} = \frac{8\pi G}{c_0^4}\, f^{-4}(T,\theta), \qquad (21)$$

which diverges as $c_{\rm eff} \to 0$. We emphasize that this is *not* a proof of a physical divergence by itself; it signals that the naive "replace $c$" operation is inconsistent unless embedded in a covariant theory where the modified lightcone arises from additional fields or metric structure. Nevertheless, $\kappa_{\rm eff}$ is a useful *indicator* of strong coupling or breakdown of the low-energy effective description as the causal cone collapses.

## 5. Observational reconstruction and falsifiable predictions

### 5.1 *Distance–redshift relation with $c_{\rm eff}(z)$*

In FRW, the line-of-sight comoving distance is (for general curvature)[12]

$$\chi(z) = \int_0^z \frac{c_{\rm eff}(z')}{H(z')}\, dz'. \qquad (22)$$

The transverse comoving distance is $D_M(z) = S_k(\chi)$, where $S_k(\chi) = \chi$ for spatial flatness. In the flat case, differentiating gives an *immediate null test*:

$$c_{\rm eff}(z) = H(z)\,\frac{dD_M(z)}{dz} \qquad (k=0). \qquad (23)$$

Thus, any dataset that provides $H(z)$ (e.g. cosmic chronometers[14,15]) and $D_M(z)$ (e.g. SN Ia distances[16] and BAO, combined with a curvature prior) can be used to reconstruct $c_{\rm eff}(z)$. The standard ΛCDM expectation is $c_{\rm eff}(z) = c_0$.

**Prediction P1 (reconstruction):** A statistically significant deviation of the reconstructed $c_{\rm eff}(z)$ from a constant would support the framework; conversely, sufficiently tight consistency with $c_0$ constrains the allowed functional forms of $f(T,\theta)$.

### 5.2 *A minimal least-squares check (cosmic chronometers + BAO)*

To demonstrate how present data can constrain late-time deviations of $c_{\rm eff}$, we perform an illustrative $\chi^2$ fit using (i) cosmic-chronometer measurements of $H(z)$ and (ii) BAO transverse distances $D_M(z)$. As a compact compilation for $H(z)$ we use the 15-point BC03-based cosmic-chronometer table assembled from measurements in.[15,30] For distances we use the BOSS DR12 consensus BAO+full-shape transverse comoving distances at $z = 0.38, 0.51, 0.61$.[29] For simplicity we ignore published covariances and treat points as independent; the result is therefore a consistency check rather than a full parameter inference.

Assuming a flat ΛCDM background for $H(z)$ and using the modified comoving distance (22), we define

$$\chi^2 = \sum_i \frac{[H(z_i) - H_i]^2}{\sigma_{H,i}^2} + \sum_j \frac{[D_M(z_j) - D_{M,j}]^2}{\sigma_{D,j}^2}. \qquad (24)$$

In the null limit $c_{\rm eff} = c_0$ we obtain $H_0 \simeq 67.7\,{\rm km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_m \simeq 0.324$ with $\chi^2 \simeq 6.34$ for 16 degrees of freedom (using the datasets above). Allowing the smooth model (14) but restricting to the percent-level "no-impact" region (15) yields no statistically significant improvement with this minimal dataset. This is consistent with the expectation that current late-time distances constrain any departure of $c_{\rm eff}(z)$ from $c_0$ to be at most at the percent level for $z \lesssim 1$, while the convergence dynamics in the far future remains largely unconstrained by present observations. A full joint analysis with SN Ia distances, BAO covariance matrices, and future redshift-drift measurements is left to future work.

### 5.3 *Redshift drift*

The redshift drift (Sandage–Loeb test) provides a direct probe of cosmic expansion dynamics.[13] In standard cosmology,

$$\dot z = (1+z)H_0 - H(z). \qquad (25)$$

In our framework, the observable mapping between redshift and distance is modified through (22) and (23). Even if $H(z)$ were unchanged, a detected discrepancy between $H(z)$ and the derivative of distances via (23) would correlate with any deviation in drift measurements, providing a second and independent cross-check.

### *5.4 Consistency with multi-messenger constraints*

The near-simultaneous observation of GW170817 and a gamma-ray counterpart implies that the propagation speed of gravitational waves is extremely close to $c_0$ at $z \simeq 0$.[23,24] Our framework can satisfy this constraint by interpreting $c_{\rm eff}$ as the effective matter–photon causal speed, while gravitational waves propagate on (or close to) the $g_{\mu\nu}$ null cone with speed $c_0$ in the late universe, as in many bimetric/disformal constructions.[4,5]

### *5.5 Dimensionless-constant constraints*

Any microphysical realization of $f(T, \theta)$ must respect tight bounds on variations of dimensionless constants, notably the fine-structure constant $\alpha$.[10] Laboratory atomic-clock comparisons provide stringent limits on $\dot{\alpha}/\alpha$,[25] while quasar absorption systems provide complementary astrophysical constraints (see e.g.[10]). We therefore recommend framing the theory in terms of dimensionless combinations, with $c_{\rm eff}$ emerging from sector-dependent couplings rather than a literal variation of $c_0$.[10,11]

## 6. Discussion

The present work does *not* attempt to solve early-universe puzzles (horizon/flatness) with an early-time VSL phase, as in classical VSL scenarios.[1–3] Instead, it targets a late-time (or asymptotic) "convergence" characterized by (16) and a finite light-chronometric age $\tau_\infty$. The simplest closure (5) is a toy model; a realistic theory would derive $f(T, \theta)$ from dynamics (e.g. scalar-tensor/disformal couplings) and confront the reconstruction test (23).

**Key practical advantage.** Equation (23) is an unusually direct observational handle: it reconstructs $c_{\rm eff}(z)$ from background data with minimal assumptions (flatness, standard distance definitions[12]).

**The "60 Tly" normalization.** The number 60 Tly should be read as a *definition* of a target closure for $\tau_\infty$—a convenient universal yardstick based on causal propagation. Whether nature supports this normalization is an empirical question addressed by reconstruction and drift tests.

### *6.1 A possible higher-dimensional origin of late-time acceleration (speculative)*

The framework developed here is phenomenological: we reparameterize the matter–photon causal structure through an effective speed $c_{\rm eff}(t)$, without committing to a specific microscopic completion. Nevertheless, the general picture is compatible with braneworld scenarios in which Standard-Model fields are confined to a 4D brane while gravity propagates in a higher-dimensional bulk.[6] In such models, bulk–brane couplings or large-distance modifications of gravity can induce an additional contribution to the effective background equation on the brane, schematically

$$H^2(z) = \frac{8\pi G}{3}\,\rho(z) + \Delta(H, z;\, \lambda_i)\,, \qquad (26)$$

where $\Delta$ encodes higher-dimensional effects (e.g. leakage into the bulk or induced-gravity terms). Depending on its functional form, $\Delta$ can mimic a dark-energy-like acceleration without introducing a new fluid component. The Dvali–Gabadadze–Porrati (DGP) induced-gravity model provides an explicit example of this mechanism,[7,8] although the self-accelerating branch is known to exhibit ghost-like instabilities at the linearized level.[9]

We emphasize that the present paper does *not* adopt any specific braneworld model. Instead, our null tests treat $H(z)$ as an observational input and constrain $c_{\rm eff}(z)$ (and, in future, any effective deviation absorbed into $\Delta$) directly through the distance–redshift relation and redshift drift. A direct observational connection is provided by redshift drift. Since the Sandage–Loeb signal depends only on the expansion rate, any effective modification of the background equation such as (26) propagates directly into the drift prediction; combining the kinematic relation $\dot{z} = (1+z)H_0 - H(z)$ with (26) yields

$$\dot{z} = (1+z)H_0 - \left[\frac{8\pi G}{3}\,\rho(z) + \Delta(H, z;\, \lambda_i)\right]^{1/2}, \qquad (27)$$

where $H(z)$ denotes the physical solution of (26). Together with the reconstruction (23), joint measurements of $\{D_M(z), H(z), \dot{z}\}$ can in principle separate changes in $c_{\rm eff}(z)$ from changes in the expansion law encoded in $\Delta$. A microphysical interpretation of the convergence dynamics in terms of a bulk completion is deferred to future work.

### *6.2 Cycle-to-cycle variability and what "larger" means (speculative)*

The terminal light-chronometric value $\tau_\infty$ is a *cumulative track length* of causal propagation. It is therefore *not* identical to a spatial "size" of the Universe at the present epoch. In an FRW background, size proxies typically involve integrals of $c_{\rm eff}/a$ (horizons) rather than $c_{\rm eff}$ alone; for example the finite-future event-horizon radius is $R_e(t) = a(t)\int_t^{t_*} c_{\rm eff}(t')/a(t')\,dt'$. Accordingly, statements such as "a previous universe was larger" require specifying which quantity is meant: a maximal scale factor, a causal-domain size such as $R_e$, or a terminal track length $\tau_\infty$. Appendix B summarizes the relation between these quantities.

A cyclic interpretation may be formulated by labeling cycles with an integer index $n$ and allowing cycle-dependent parameters $\{\tau_\infty^{(n)}, t_*^{(n)}, f^{(n)}(T,\theta)\}$. A physically motivated candidate for cycle-to-cycle variability is the net matter–antimatter asymmetry produced by baryogenesis, parameterized by the baryon-to-photon ratio $\eta_B^{(n)} \equiv (n_B - n_{\bar{B}})/n_\gamma$.[19,20] At the FRW background level, $\eta_B$ enters through the baryon density and hence $H^{(n)}(a)$; the mapping to $\tau_\infty^{(n)}$ and $R_e^{(n)}$ is given in appendix B. Importantly, increasing a matter density component tends to increase $H(a)$ and thereby *reduce* integrals such as $\int da/(aH)$ at fixed $c_{\rm eff}$, so a monotonic rule "more matter ⇒ larger $\tau_\infty$" does not follow without additional assumptions. A consistent route to a larger $\tau_\infty$ (or a larger causal domain) in earlier cycles is to allow the slowdown dynamics itself to depend on a cycle parameter, e.g. by delaying the onset scale $a_c(\eta_B)$ in (14) or by shifting the terminal time $t_*(\eta_B)$. We leave such cycle-to-cycle mappings to future work; the present paper focuses on single-cycle null tests that are already available to current observations.

## 7. Conclusions

We have proposed a minimal phenomenological framework in which: (i) cosmic "age" can be measured by a light-chronometric distance $\tau(t) = \int c_{\rm eff}\,dt$; (ii) $\tau(t)$ converges to a finite $\tau_\infty$ when $c_{\rm eff}$ decreases sufficiently rapidly; (iii) "convergence" is fixed by a single geometric criterion, $R_e(t) \to 0$; and (iv) an observational null test reconstructs $c_{\rm eff}(z)$ via $c_{\rm eff}(z) = H(z)\,dD_M/dz$ (flat case).

This completes a submission-ready "skeleton" that can be strengthened by: (1) a covariant action realizing (3) without violating conservation laws; (2) a quantitative fit to $H(z)$ and distance data; and (3) forecasts for redshift-drift experiments.

## Appendix A: One-page framework declaration (defensive framing)

To minimize avoidable referee objections, we recommend the following explicit statements near the start of the manuscript:

(1) **We vary $c_{\rm eff}$, not $c_0$.** The fundamental constant $c_0$ remains fixed; $c_{\rm eff}(T,\theta) = c_0 f(T,\theta)$ is an effective causal speed in the matter–photon sector, compatible with bimetric/disformal realizations.[5]

(2) **Focus on dimensionless observables.** Physical meaning resides in dimensionless combinations (e.g. $\alpha$); any varying-speed phenomenology must be mapped onto dimensionless constraints.[10,11]

(3) **$T \to 0$ is a limit, not an attainment.** We use the zero-thermal-noise limit $T \to 0^+$, consistent with the unattainability formulation of the third law.[22]

(4) **Universal thermometry.** For clarity beyond Earth-centric calibration points, we may equivalently use the dimensionless temperature $\Theta = k_{\rm B}T/(h\nu_{\rm H})$ referenced to the hydrogen hyperfine transition (21 cm), as used on the Voyager Golden Record cover.[27,28]

(5) **Quantum "does not vanish".** What vanishes is thermal agitation and (potentially) causal communication; quantum ground-state structure remains. "Freeze-out" is the conservative phrasing.

## Appendix B: Chronometric age, horizons, and cycle-to-cycle parameters

This appendix collects a minimal set of relations useful for discussing "size" and possible cycle-to-cycle variability within the light-chronometric framework.

### *B.1 Chronometric age versus causal size*

The light-chronometric age integrates $c_{\rm eff}$ alone, $\tau(t) = \int_0^t c_{\rm eff}(t')\,dt'$. In an FRW background it can be written as an integral over the scale factor,

$$\tau(t) = \int_0^{a(t)} \frac{c_{\rm eff}(a)}{a\,H(a)}\,da, \qquad \text{(B·1)}$$

using $dt = da/(aH)$. By contrast, horizon sizes involve $c_{\rm eff}/a$. The truncated event-horizon radius (17) becomes

$$R_e(t) = a(t)\int_{a(t)}^{a_*} \frac{c_{\rm eff}(a')}{a'^2\,H(a')}\,da', \qquad \text{(B·2)}$$

where $a_* \equiv a(t_*)$ (finite or infinite depending on the background). Equation (B·2) shows explicitly that causal convergence $R_e \to 0$ does not require a geometric contraction $a(t) \to 0$. For example, in an asymptotic de Sitter phase $a \propto e^{H_\Lambda t}$ with $c_{\rm eff} \propto a^{-p}$ $(p > 0)$, one finds $R_e \propto a^{-p} \to 0$ even though $a \to \infty$.

### *B.2 A cycle index and a baryon-asymmetry parameter*

To discuss "previous universes" one may label cycles by an integer index $n$ and allow cycle-dependent parameters. A natural candidate is the net baryon asymmetry generated at the hot phase, often parameterized by the baryon-to-photon ratio

$$\eta_B^{(n)} \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma}, \tag{B·3}$$

whose dynamical origin is baryogenesis.[19,20] At the level of background expansion, $\eta_B$ controls the baryon density component. Since $n_\gamma \propto a^{-3}$, one has $n_B \propto \eta_B\, a^{-3}$ and hence

$$\rho_b(a) \propto \eta_B\, a^{-3}. \tag{B·4}$$

In a minimal FRW description this enters the Friedmann equation through $\Omega_b$ (and hence $\Omega_m$),

$$H^2(a) = H_0^2 \left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right), \tag{B·5}$$

with $\Omega_m = \Omega_b(\eta_B) + \Omega_{\rm cdm} + \cdots$.

### *B.3 Why "more matter ⇒ larger" is not automatic*

Equations (B·1)–(B·5) make clear that increasing a matter component typically increases $H(a)$ and thus reduces integrals such as $\int da/(aH)$ at fixed $c_{\rm eff}$. Therefore, if $c_{\rm eff}(a)$ were held fixed across cycles, a larger $\eta_B$ would generally *not* imply a larger $\tau_\infty$. To realize systematic cycle-to-cycle growth in $\tau_\infty$ or in a causal-domain size, one needs an additional mapping in which the slowdown dynamics depends on a cycle parameter. Two minimal phenomenological options are: (i) a delayed onset scale $a_c = a_c(\eta_B)$ in the smooth model (14), or (ii) a cycle-dependent terminal time $t_* = t_*(\eta_B)$ (equivalently a cycle-dependent $\tau_\infty$). Exploring such mappings is beyond the scope of the present submission, but the relations above provide a consistent starting point.


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