

Geometric and Causal Drift in Cyclic Cosmology: A Phenomenological Classification from Extra Spatial Freedom, Baryon Asymmetry, and Memory

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Abstract

We formulate a second-step phenomenology for cyclic cosmology in which the main question is not whether successive cosmic cycles *must* become larger or smaller, but under which conditions they do so. The framework distinguishes *geometric size* and *causal size*, since cyclic scenarios in the literature already show that an increase of the scale factor from one cycle to the next need not imply an increase of the causal domain, nor of local bang-scale observables. For each cycle n , we introduce a minimal “cycle genome”

$$g_n = (\chi_n, \eta_{B,n}, \varepsilon_n),$$

where χ_n parametrizes effective extra spatial freedom, $\eta_{B,n}$ is baryon asymmetry, and ε_n is inter-cycle memory (or residual copying noise). We then define a drift map

$$\varepsilon_{n+1} = q\varepsilon_n, \quad \chi_{n+1} = \chi_n + \nu\varepsilon_n, \quad \eta_{B,n+1} = \eta_{B,n}(1 + s\kappa\varepsilon_n),$$

with $0 < q < 1$, $\kappa > 0$, and $s = \pm 1$. Any macroscopic cycle quantity Q_n (normalized geometric size, causal size, bang energy scale, or light-chronometric lifetime) is then written as

$$Q_n = Q_0 \exp(\gamma_Q \chi_n) \left(\frac{\eta_{B,n}}{\eta_{B,0}} \right)^{\delta_Q}.$$

To first order in memory, the sign of a single coefficient,

$$\Xi_Q \equiv \gamma_Q \nu + \delta_Q s \kappa,$$

classifies whether Q_n grows, decays, or remains stationary from cycle to cycle. The result provides a compact language for discussing whether earlier universes were geometrically larger or smaller, whether their causal domains were broader or narrower, and whether bang-scale energy increased or decreased. The construction is self-contained, all figures are embedded in the main text, and black holes are used only as limit cases of classical geometric breakdown rather than as evidence for extra dimensions.

1 Introduction

Cyclic cosmology is not a single theory but a family of scenarios in which expansion, contraction, bounce, or conformal crossover recur in some effective sense. Different realizations already exhibit qualitatively different inter-cycle behavior. In entropy-driven cyclic pictures, cycle size and lifetime may grow from one cycle to the next; in ekpyrotic and newer non-singular cyclic models, local thermodynamic quantities may repeat while the overall scale factor keeps increasing; and recent work has clarified that entropy arguments alone do not force a unique verdict on geodesic completeness or on the physical relevance of cycle-to-cycle rescaling [1, 2, 3, 4, 5]. This diversity suggests that a useful second-step phenomenology should not begin by assuming that earlier cycles were necessarily larger or smaller. Instead, it should identify the minimal variables that determine *when* geometric size, causal size, or bang-scale observables drift upward or downward.

The present paper develops such a classification. Its starting point is intentionally modest. We do not attempt to derive a full microphysical bounce model. We also do not remove time

from general relativity. The ambient spacetime is still treated in the standard 3 + 1 language. If additional dimensions exist, they are taken here only as *additional spatial freedom*, summarized phenomenologically by an effective parameter rather than invoked as directly observed directions. Black holes likewise enter only as *limit cases*: they are reminders that classical geometric description can fail, not proofs of extra-dimensional structure.

The paper focuses on four macroscopic questions:

- (i) Does the *geometric size* of successive cycles increase or decrease?
- (ii) Does the *causal size* of successive cycles increase or decrease?
- (iii) Does the bang-scale energy (or temperature) increase or decrease?
- (iv) Which hidden inter-cycle variables control these drifts?

The contributions of this note are compact:

- (a) it separates *geometric* and *causal* notions of cycle size;
- (b) it introduces a three-parameter cycle genome and a minimal drift map;
- (c) it derives a sign-classification rule that decides, quantity by quantity, whether inter-cycle drift is upward, downward, or absent;
- (d) it embeds the corresponding visual diagnostics directly in the main text.

The answer proposed here is a minimal “cycle genome”

$$g_n = (\chi_n, \eta_{B,n}, \varepsilon_n), \quad (1)$$

where χ_n measures the effective participation of extra spatial freedom, $\eta_{B,n}$ is the baryon asymmetry parameter familiar from baryogenesis [6, 7], and ε_n measures the residual memory that survives one cycle-to-cycle transfer. The main result is that, once macroscopic quantities are written as simple functions of $(\chi_n, \eta_{B,n})$, a single sign coefficient determines whether any chosen quantity grows or decays from cycle to cycle.

In that sense, this paper is not a claim that the previous universe was definitely larger or definitely smaller. It is a *classification theorem*: it states the conditions under which either conclusion follows.

2 Geometric size, causal size, and the cycle genome

2.1 Why geometric size and causal size must be separated

A central point of the present framework is that “size” is not unique in cyclic cosmology. We therefore distinguish:

- **Geometric size**, represented by a normalized scale ratio

$$\mathcal{A}_n \equiv \frac{a_{\max,n}}{a_{b,n}}, \quad (2)$$

where $a_{\max,n}$ is a characteristic maximal scale factor within cycle n and $a_{b,n}$ is a cycle-internal reference value (for example, the bounce scale or another natural normalization point).

- **Causal size**, represented by a characteristic horizon length $R_{c,\max}^{(n)}$. When a future event horizon exists during a cycle, we identify R_c with the finite-future event-horizon scale. When such a horizon is absent, we instead use the apparent horizon / Hubble horizon, which exists generically in FRW cosmology [9].

This separation is not cosmetic. Existing cyclic models already demonstrate that the overall scale factor can grow from one cycle to the next while local temperature or density re-enter nearly the same range, or while the relevant causal domain is reset by horizon structure [1, 3, 5]. A viable classification of inter-cycle evolution must therefore allow geometric and causal sizes to drift independently.

2.2 Cycle genome

We parameterize each cycle by the triplet in Eq. (1):

- χ_n — *extra spatial freedom*, a phenomenological variable summarizing how strongly additional spatial degrees of freedom (if any) participate in the effective geometry. It does not claim direct observability. Rather, it is a bookkeeping parameter that may encode compactification, localization, leakage, or other geometric effects familiar from higher-dimensional settings [8].
- $\eta_{B,n}$ — *baryon asymmetry*, defined in the usual way by the baryon-to-photon asymmetry. We use it as an inter-cycle state variable, not as a new baryogenesis mechanism.
- ε_n — *memory* or *residual copying noise*, representing the small mismatch that survives cycle-to-cycle reset. The framework is built so that ε_n decays, i.e. the system tends toward a fixed-point-like regime rather than drifting without bound.

3 Minimal drift map

3.1 Definition

The smallest nontrivial inter-cycle drift map is

$$\varepsilon_{n+1} = q \varepsilon_n, \quad 0 < q < 1, \quad (3)$$

$$\chi_{n+1} = \chi_n + \nu \varepsilon_n, \quad (4)$$

$$\eta_{B,n+1} = \eta_{B,n} (1 + s \kappa \varepsilon_n), \quad s = \pm 1, \kappa > 0. \quad (5)$$

The interpretation is straightforward. The memory variable decays geometrically. The extra-spatial parameter χ_n shifts by an amount proportional to the residual memory. The baryon asymmetry drifts multiplicatively, with $s = +1$ or $s = -1$ controlling the sign of the drift.

The exact solution of Eq. (3) is

$$\varepsilon_n = \varepsilon_0 q^n. \quad (6)$$

For $n \geq 0$, Eq. (4) gives

$$\chi_n = \chi_0 + \nu S_n, \quad S_n \equiv \sum_{j=0}^{n-1} \varepsilon_j = \varepsilon_0 \frac{1 - q^n}{1 - q}. \quad (7)$$

Similarly,

$$\eta_{B,n} = \eta_{B,0} \prod_{j=0}^{n-1} (1 + s \kappa \varepsilon_0 q^j). \quad (8)$$

When $\kappa\varepsilon_n \ll 1$, the logarithm linearizes to

$$\ln \frac{\eta_{B,n}}{\eta_{B,0}} = s\kappa S_n + \mathcal{O}(\varepsilon_0^2). \quad (9)$$

3.2 Macroscopic cycle quantities

We now assume that any cycle quantity of interest can be represented phenomenologically as

$$Q_n = Q_0 \exp(\gamma_Q \chi_n) \left(\frac{\eta_{B,n}}{\eta_{B,0}} \right)^{\delta_Q}, \quad (10)$$

where γ_Q and δ_Q are coupling exponents determined by the chosen effective model. The framework does not claim that these exponents are universal constants; they summarize how a given macroscopic quantity responds to the two inter-cycle state variables.

Important examples are:

$$Q_n = \mathcal{A}_n, \quad Q_n = R_{c,\max}^{(n)}, \quad Q_n = \rho_{\max,n}, \quad Q_n = \tau_\infty^{(n)}. \quad (11)$$

That is, one and the same formalism can classify drift in normalized geometric size, causal size, bang energy scale, and light-chronometric cycle lifetime.

4 Sign-classification theorem

Substituting Eqs. (7) and (9) into Eq. (10) gives, to first order in memory,

$$\ln \frac{Q_n}{Q_0} = (\gamma_Q \nu + \delta_Q s\kappa) S_n + \mathcal{O}(\varepsilon_0^2). \quad (12)$$

Define the *drift classifier*

$$\Xi_Q \equiv \gamma_Q \nu + \delta_Q s\kappa. \quad (13)$$

Proposition 1 (First-order sign classification). *Assume $\kappa\varepsilon_n \ll 1$ for the cycles of interest. Then, for $n \geq 0$,*

$$\ln \frac{Q_{n+1}}{Q_n} = \Xi_Q \varepsilon_n + \mathcal{O}(\varepsilon_n^2). \quad (14)$$

Hence:

- if $\Xi_Q > 0$, then Q_n increases monotonically from cycle to cycle to first order;
- if $\Xi_Q < 0$, then Q_n decreases monotonically from cycle to cycle to first order;
- if $\Xi_Q = 0$, then there is no first-order drift.

This is the central result of the paper. It immediately answers the motivating questions:

- Earlier universes were geometrically larger than the current one if $\Xi_{\mathcal{A}} < 0$.
- Earlier universes had larger causal domains if $\Xi_{R_c} < 0$.
- Earlier bang-scale energies were larger if $\Xi_\rho < 0$ toward the future (equivalently, larger toward the past).

The framework therefore converts qualitative cosmological language into a sign problem.

Table 1: A representative parameter choice used in the illustrative figures. The point of the table is not to claim these coefficients are unique, but to show how one set of couplings can produce opposite drift directions for geometric and causal size.

Quantity Q_n	γ_Q	δ_Q	Ξ_Q	Trend (future)
\mathcal{A}_n	1.0	0.40	+0.52	grows
$R_{c,\max}^{(n)}$	-0.60	0.20	-0.62	shrinks
$\rho_{\max,n}$	-0.30	1.10	-1.01	shrinks

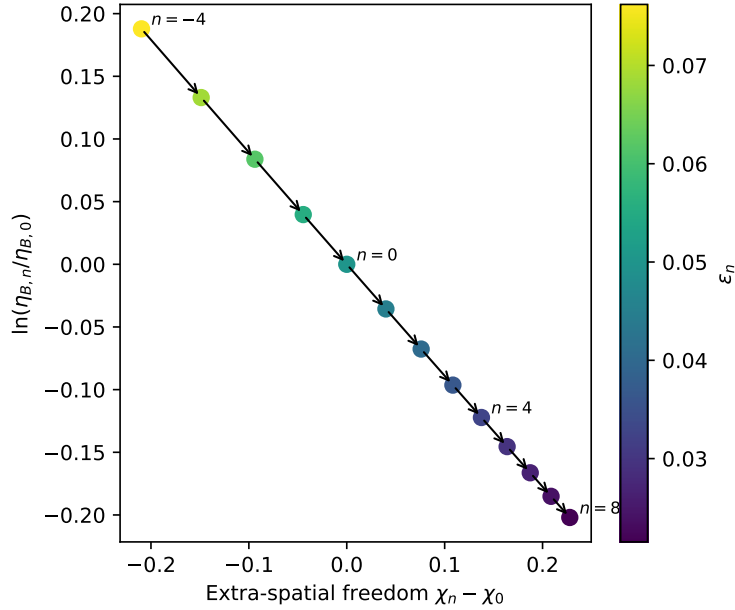


Figure 1: Illustrative inter-cycle trajectory in the cycle-genome plane $(\chi_n - \chi_0, \ln(\eta_{B,n}/\eta_{B,0}))$. Arrows indicate increasing cycle number and the color scale represents the decaying memory ε_n . The sample parameters are $\varepsilon_0 = 0.05$, $q = 0.90$, $\nu = 0.80$, $\kappa = 0.70$, and $s = -1$.

5 Visual phenomenology

The strength of the present classification is that its content is naturally visual. Rather than asking the reader to imagine how different cycle variables move, one can draw the state-space flow, the quantity trajectories, and the sign-classification quadrants directly.

5.1 Cycle-genome flow

Figure 1 shows a sample trajectory in the $(\chi_n, \ln \eta_{B,n})$ plane. The arrows follow increasing cycle number, while the color indicates the decaying memory variable ε_n . Because both drifts are proportional to ε_n , the flow naturally slows as the sequence approaches the fixed regime.

5.2 A four-quadrant classification of size drift

Figure 2 is the most compressed summary of the theory. The horizontal axis is $\Xi_{\mathcal{A}}$ and the vertical axis is Ξ_{R_c} . The four quadrants correspond to the four logically possible combinations of geometric and causal drift. In particular, the fourth quadrant demonstrates a possibility that is often obscured in verbal discussions: a universe may become geometrically larger from cycle

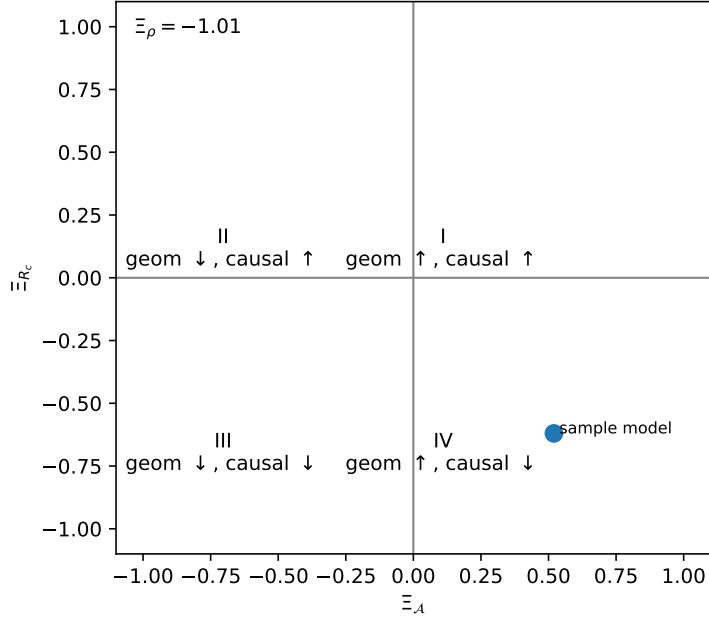


Figure 2: Four-quadrant classification in the plane $(\Xi_{\mathcal{A}}, \Xi_{R_c})$. The plotted point is the sample parameter choice of Table 1. A single model can therefore lie in a regime where geometric size grows while causal size shrinks. The annotation reports the sample value of Ξ_{ρ} .

to cycle while its causal domain becomes smaller.

5.3 Illustrative trajectories

Figure 3 shows the sample evolution of normalized geometric size, causal size, and bang energy scale across cycle number. The curves are not intended as observational fits. Their purpose is to make the sign structure tangible. In the sample model, future cycles are geometrically larger, but causally narrower and energetically weaker. Earlier cycles are therefore smaller in geometric size but larger in causal size and bang-scale energy.

Figure 4 shows the relation between geometric size and bang energy scale in the same sample model. The quantity pair obeys an approximately linear relation in log–log form,

$$\ln \frac{\rho_{\max,n}}{\rho_{\max,0}} \approx \frac{\Xi_{\rho}}{\Xi_{\mathcal{A}}} \ln \frac{\mathcal{A}_n}{\mathcal{A}_0}, \quad (15)$$

provided $\Xi_{\mathcal{A}} \neq 0$. This offers a compact way to discuss whether “larger” universes are associated with stronger or weaker bangs in a chosen drift model.

6 Physical reading and caveats

6.1 What the model does and does not claim

The present paper is a phenomenological classifier, not a completed microphysical theory. It does *not* claim:

- that χ_n has been directly measured;
- that baryogenesis across cycles is solved;

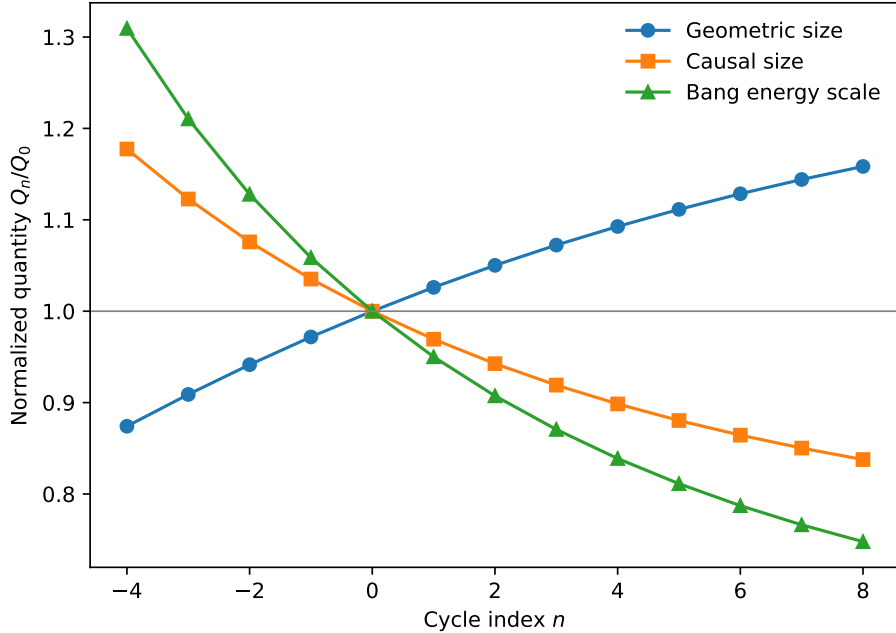


Figure 3: Illustrative normalized trajectories for geometric size, causal size, and bang energy scale, generated with the sample parameters of Table 1. The present cycle is indexed by $n = 0$. The figure is a visualization of the sign classification, not an observational fit.

- that black holes prove extra dimensions;
- that one specific cyclic scenario is singled out by data.

Instead, it offers a compact language in which different cyclic mechanisms can be compared. If a more microscopic model predicts how χ_n , $\eta_{B,n}$, and ε_n transform, then the present classifier immediately states how geometric size, causal size, and bang-scale observables must drift.

6.2 Black holes as limit cases, not evidence

Black holes motivate the framework only in a limited sense. They provide familiar examples of regimes where classical geometric description becomes singular or horizon-dependent, and where naive intuition about space and time becomes unreliable. In the present paper they are used solely as reminders that geometric variables can fail or change character in extreme regimes. They are *not* taken as empirical proof that χ_n represents actual observed extra dimensions.

6.3 Relation to higher-dimensional intuition

If one wishes to attach a geometric interpretation to χ_n , the most conservative reading is this: χ_n measures how strongly effective *extra spatial freedom* participates in the cycle-to-cycle reconstruction of the universe. This language is compatible with higher-dimensional intuitions familiar from brane or bulk scenarios, while remaining agnostic about the actual microscopic realization [8]. The virtue of this phrasing is that it keeps the paper inside a phenomenological lane.

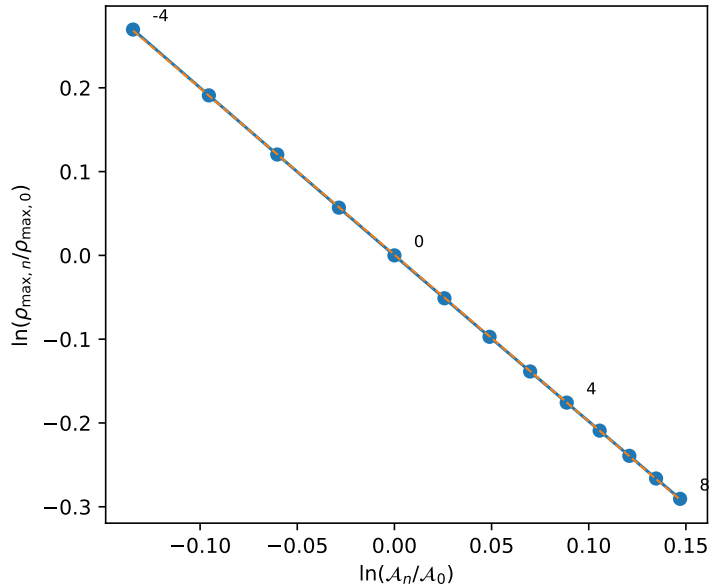


Figure 4: Bang energy scale versus geometric size in the sample model. Cycle labels indicate the direction of drift relative to the present cycle $n = 0$. The slope is determined by $\Xi_\rho/\Xi_{\mathcal{A}}$.

7 Conclusion

The second-step problem in cyclic cosmology is not merely whether the universe repeats. It is whether successive cycles drift, and if so in what direction. We have shown that a minimal three-parameter cycle genome,

$$g_n = (\chi_n, \eta_{B,n}, \varepsilon_n),$$

together with a simple drift map and the macroscopic parameterization in Eq. (10), is already sufficient to classify the cycle-to-cycle evolution of normalized geometric size, causal size, bang energy scale, and light-chronometric lifetime.

The main result is the sign classifier

$$\Xi_Q = \gamma_Q \nu + \delta_Q s \kappa,$$

which determines, to first order in memory, whether a chosen quantity Q_n grows, decays, or remains stationary across cycles. This isolates the core theoretical question: not *which* cyclic story one prefers, but which microscopic assumptions force the sign of Ξ_Q for the quantities of interest.

The framework naturally accommodates the possibility that geometric size and causal size drift in opposite directions. That possibility, already foreshadowed in parts of the cyclic literature, becomes especially clear once the two notions of size are kept separate. In this language, the universe may become geometrically larger while becoming causally narrower. In that sense, the cycle is not a simple repetition but a phase-shifted recurrence. The next step is to connect concrete bounce or braneworld constructions to the coefficients (γ_Q, δ_Q) , and eventually to ask whether any inter-cycle drift leaves an observable imprint within a single cycle.

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