Chronometric Geometry in Cyclic Cosmology: Extra Spatial Freedom and an Infinite Sequence of Cycles

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Abstract

We present a third-step phenomenological framework that closes a sequence of companion papers on cyclic cosmology. Standard 3+1 spacetime is retained, but no additional timelike dimensions are introduced. All extra degrees of freedom are treated as spatial, and cosmological evolution is encoded by a monotonic chronometric parameter rather than by reversible motion along an added time axis. In this sense, higher-dimensionality is defined operationally as the presence of one or more extra spatial degrees of freedom beyond the observed three spatial directions, $D = 4 + d_s$ with $d_s \geq 1$, without committing to a specific ultraviolet completion such as a fixed 10- or 11-dimensional model.

The paper clarifies the geometric meaning of the light-chronometric construction of Paper I and the inter-cycle drift model of Paper II. In particular, the effective causal speed c_{eff} introduced previously is interpreted as a sector-dependent chronometric quantity, not as a variation of the fundamental vacuum constant c_0. A future-oriented evolution law, $d\tau/dt > 0$, together with a cycle index $n \to n + 1$, yields an infinite sequence of cycles rather than an infinite timelike loop. Within this viewpoint, black holes are used only as limit cases where classical geometric description becomes extreme or incomplete, not as direct evidence for extra dimensions.

The resulting framework provides a conceptually coherent closure of the previous two papers: Paper I supplied a finite light-chronometric age and a causal convergence condition, Paper II supplied inter-cycle drift, and the present work supplies the geometric interpretation that separates extra spatial freedom from timelike extension.

1 Introduction

The standard relativistic description of the universe is formulated in terms of 3+1 spacetime, where three coordinates describe space and one coordinate describes time. At the same time, many extensions of fundamental physics introduce additional dimensions, often motivated by ultraviolet completion, compactification, or higher-dimensional gravity [3, 4]. In such discussions, however, the word "time" is frequently overloaded: it serves as a coordinate in spacetime geometry, as an ordering parameter for physical change, and as an intuitive label for irreversible evolution. The present work argues that these roles need not be conflated.

Our starting point is conservative. We do not remove time from standard spacetime geometry, nor do we deny the 3+1 structure of general relativity. Instead, we impose a narrower hypothesis: no additional timelike dimensions are introduced beyond the conventional relativistic one. All extra degrees of freedom considered here are spatial. Cosmological evolution is then described not as reversible motion along an added time axis, but by a monotonic chronometric order. This shift is motivated by the structure already developed in our previous two companion manuscripts [1, 2].

In Paper I, cosmic age was redefined through a light-chronometric quantity,

$$\tau(t) = \int_0^t c_{\mathrm{eff}}(t')\, dt', \tag{1}$$

where c_{eff} denotes an effective causal propagation speed for the observable sector [1]. In that framework, the terminal state was characterized by finite light-chronometric age together with

a causal convergence condition. In Paper II, the construction was extended from intra-cycle evolution to inter-cycle drift, allowing geometric size and causal size to evolve differently from cycle to cycle [2]. That analysis suggested that cosmological recurrence need not be a simple repetition; instead, it may be understood as a phase-shifted recurrence.

The present paper provides the geometric interpretation that was still missing. We define higher-dimensionality operationally as the presence of one or more extra spatial degrees of freedom beyond the observed three spatial directions, namely $D = 4 + d_s$ with $d_s \geq 1$. This definition is intentionally model-independent: we do not commit to a specific total dimensionality such as 10 or 11, nor do we assume a particular string- or brane-theoretic completion. What matters phenomenologically is not the microscopic label of the extra dimensions, but their effective contribution to cycle evolution and causal structure.

The purpose of this paper is threefold. First, to formulate a clear operational definition of higher-dimensionality in the present phenomenological program. Second, to distinguish extra spatial freedom from timelike extension. Third, to show that the light-chronometric and inter-cycle drift constructions can be understood as parts of a single chronometric-geometric framework. In this sense, the present paper serves as a conceptual closure of the previous two papers while also preparing the way for future work on the detailed structure of cycle memory and extra spatial freedom.

2 Operational framework: standard spacetime retained, timelike extension restricted

We retain the standard relativistic picture in which the observed universe is described by one timelike and three spacelike directions. If extra freedom exists, we restrict it to be spatial. Operationally, we write

$$D = 4 + d_s, \qquad d_s \geq 1, \tag{2}$$

with metric signature

$$\mathrm{sig}(g_{AB}) = (-, +, +, +, \underbrace{+, \ldots, +}_{d_s}), \tag{3}$$

so that no additional timelike directions are introduced.

This restriction is important for the logic of the framework. If extra timelike axes were admitted, one would have to address reversible motion, loop-like temporal interpretation, and the possibility of closed timelike constructions. None of these is assumed in the present program. The working hypothesis is simpler: observed physics is ordered by one standard relativistic time coordinate, while all additional freedom enters through spatial geometry and its effective coupling to observable sectors.

To keep the notation compact, we summarize the roles of the main variables in Table 1. The table is not a new law of nature; it is an operational dictionary for the present phenomenology.

3 Monotonic chronometric order and extra spatial freedom

A central claim of this work is that cosmological evolution should be represented by an ordered and monotonic chronometric parameter rather than by additional timelike directions. In the present framework, the effective causal speed of Paper I is written as

$$c_{\mathrm{eff}}(T, \theta, \ldots) = c_0\, f(T, \theta, \ldots), \qquad 0 < f \leq 1, \tag{4}$$

Table 1: Operational roles of the main variables in the chronometric-geometric framework.

Symbol	Operational meaning
t	Standard relativistic time coordinate used inside 3+1 spacetime.
τ	Monotonic chronometric parameter defined from an effective causal track length.
n	Cycle index labeling the ordered sequence of cycles.
d_s	Number of extra *spatial* degrees of freedom beyond the observed three spatial directions.
χ_n	Effective cycle variable encoding the cumulative influence of extra spatial freedom.
$\eta_{B,n}$	Baryon asymmetry associated with cycle n.
ε_n	Inter-cycle memory or residual copying noise.

where c_0 is the standard vacuum constant and f is a dimensionless state function of the observable sector. The corresponding chronometric parameter is then

$$\tau(t) = \int_0^t c_{\text{eff}}(t')\, dt'. \tag{5}$$

By construction,

$$\frac{d\tau}{dt} = c_{\text{eff}}(t) > 0 \qquad (0 < t < t_*), \tag{6}$$

so the evolution considered here is future-oriented, monotonic, and non-reversible.

Equation (6) is not a denial of relativistic time. Rather, it expresses that the physically relevant ordering of cosmological states in this phenomenology is not interpreted as free traversal along extra time axes. In this sense, the proposal is stronger than a mere choice of coordinates but weaker than a rejection of standard spacetime. The framework retains 3+1 relativity while forbidding timelike extension beyond it.

The low-temperature or zero-thermal-noise limit should also be read carefully. In Paper I, the behavior $c_{\text{eff}} \to 0^+$ as $T \to 0^+$ was introduced phenomenologically as a description of the observable sector. The present paper reinterprets that statement as an effective chronometric limit, not as a claim that the fundamental vacuum constant c_0 itself changes. Thus the first paper is understood here as an overture in effective chronometric geometry rather than as a literal modification of vacuum relativity.

Figure 1 summarizes the operational structure adopted in this paper.

4 An infinite sequence of cycles, not an infinite timelike loop

The cyclic aspect of the model is represented by a cycle index

$$n \in \mathbb{Z}, \qquad n \to n + 1, \tag{7}$$

which provides an ordered sequence of cycles. The key point is conceptual: cyclicity is not modeled as a closed timelike loop. It is instead an infinite sequence of ordered recurrences. The recurrence may be phase shifted, because each cycle can carry forward a small but nonzero genome-like memory from earlier cycles.

To connect with Paper II, we denote the cycle genome by

$$g_n = (\chi_n, \eta_{B,n}, \varepsilon_n), \tag{8}$$

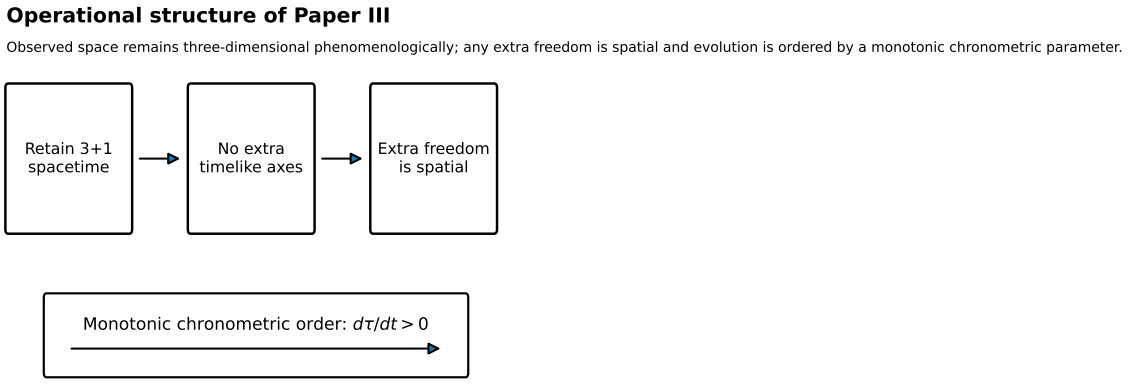

Figure 1: Operational structure of Paper III. Standard 3+1 spacetime is retained, no additional timelike axes are introduced, all extra freedom is spatial, and evolution is ordered by a monotonic chronometric parameter.

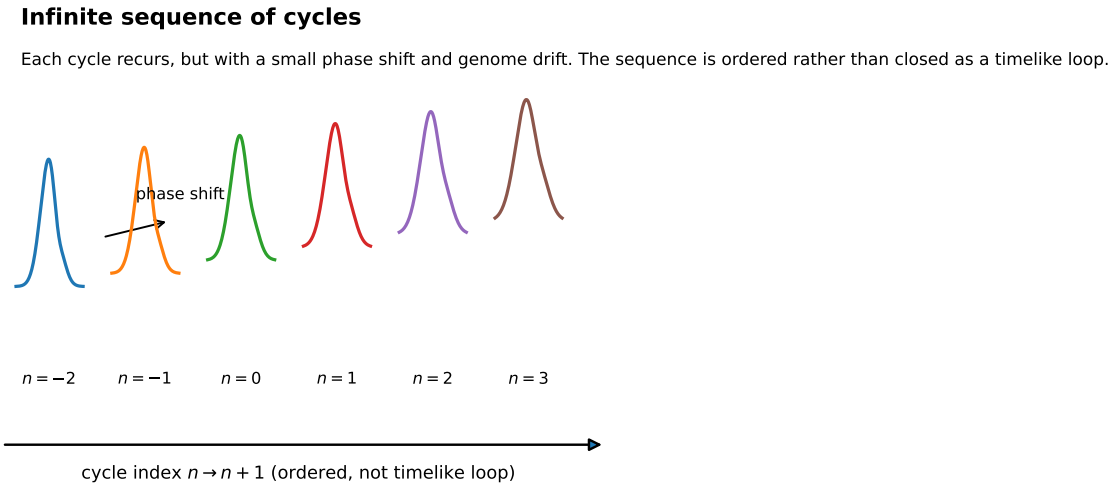

Figure 2: Infinite sequence of cycles. The recurrence is ordered by the cycle index $n \to n+1$ and may be phase shifted by a nonzero cycle genome, rather than closing into a timelike loop.

where χ_n measures the cumulative influence of extra spatial freedom, $\eta_{B,n}$ is baryon asymmetry, and ε_n is memory or copying noise. The simplest drift map is

$$\varepsilon_{n+1} = q\varepsilon_n, \qquad \chi_{n+1} = \chi_n + \nu\varepsilon_n, \qquad \eta_{B,n+1} = \eta_{B,n}(1 + s\kappa\varepsilon_n), \tag{9}$$

with $0 < q < 1$, $\kappa > 0$, and $s = \pm 1$. This map is not introduced to force a particular cosmic history; its purpose is to express mathematically that recurrence can be ordered, monotonic, and slightly phase shifted.

The resulting picture is shown schematically in Fig. 2. The sequence is infinite in the sense of ordered continuation, not in the sense of a single loop in timelike geometry.

5 Geometric size and causal size are distinct cycle observables

The second paper introduced a distinction between geometric size and causal size. The present paper shows that this distinction is the natural consequence of replacing timelike extension with

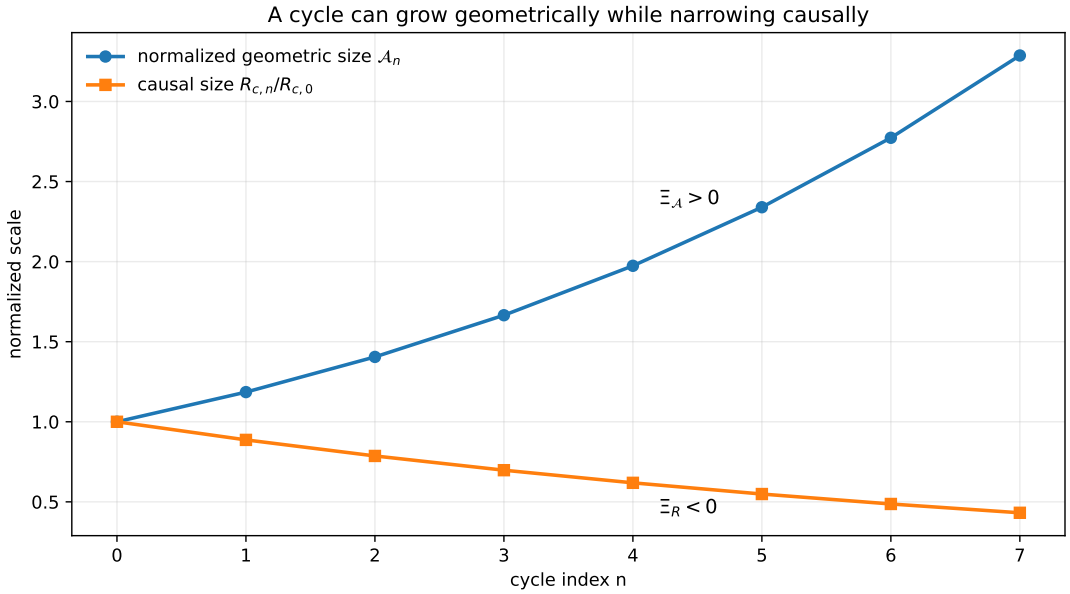

Figure 3: Illustrative drift in which normalized geometric size increases while causal size decreases. This is the simplest visual signature of a phase-shifted recurrence rather than a simple repetition.

monotonic chronometric order plus extra spatial freedom. For a cycle labeled by n, we use the normalized geometric size

$$\mathcal{A}_n \equiv \frac{a_{\max,n}}{a_{b,n}}, \tag{10}$$

and a causal size variable $R_{c,n}$, interpreted as an event-horizon-type size when such a quantity exists and as an apparent- or Hubble-horizon-type size when it does not.

The phenomenological drift law of Paper II may be written abstractly as

$$Q_n = Q_0 \exp(\gamma_Q \chi_n) \left(\frac{\eta_{B,n}}{\eta_{B,0}}\right)^{\delta_Q}, \tag{11}$$

for any cycle quantity Q_n. To first order in ε_n, the sign of

$$\Xi_Q \equiv \gamma_Q \nu + \delta_Q s \kappa \tag{12}$$

classifies whether Q_n grows or decays from cycle to cycle [2]. The important conceptual consequence is that $\Xi_{\mathcal{A}}$ and Ξ_{R_c} need not have the same sign. Therefore, a cycle can become geometrically larger while becoming causally narrower.

This case is plotted schematically in Fig. 3. It expresses in one picture the intuitive sentence that will also close the paper: the universe may become geometrically larger while becoming causally narrower.

6 Black holes as limit cases of geometric breakdown

Black holes are useful in the present context only as limit cases. They are not treated as direct evidence for higher-dimensionality. Rather, they illustrate situations in which classical spacetime description becomes extreme, ambiguous, or incomplete. In such regimes, the ordinary separation between spatial geometry, causal access, and chronometric ordering becomes conceptually stressed.

This makes black holes relevant to the present framework in a restricted way. They signal that classical geometric language has a boundary of validity, and hence motivate caution when extrapolating ordinary coordinate intuition into extreme conditions. However, they do not by themselves prove the existence of extra spatial freedom, and we do not use them as such.

7 Discussion

The present framework may be read as the conceptual closure of the previous two papers. Paper I introduced a finite light-chronometric age and a causal convergence condition. Paper II introduced inter-cycle drift and the separation between geometric size and causal size. The current paper explains why those constructions can be held together consistently: the additional structure required by the phenomenology need not be timelike. It can be entirely spatial, while cosmological evolution remains ordered by a monotonic chronometric parameter.

This viewpoint also clarifies how to speak about higher-dimensionality without overcommitting to a fixed microscopic theory. In this work, "higher-dimensional" means only the presence of one or more extra spatial degrees of freedom beyond the observed three spatial directions. We do not claim a specific total dimension such as 10 or 11; those belong to particular ultraviolet completions and are beyond the scope of the present phenomenological program.

A second advantage is interpretive economy. Once one separates geometric size from causal size, and separates extra spatial freedom from timelike extension, the cyclic picture becomes easier to state. One no longer needs to describe the universe as moving around a giant timelike loop. Instead, the sequence of cycles is modeled as an ordered chain, each element inheriting a small genome-like drift from the previous one. The result is a recurrence with memory rather than a literal temporal circle.

Future work may ask whether the effective variable χ_n should be understood as a proxy for compactification, localization, bulk leakage, or some combination of them. It may also ask whether there is a more microscopic route from the effective genome $(\chi_n, \eta_{B,n}, \varepsilon_n)$ to observational remnants in the present cycle. Those questions are postponed. The present goal is narrower: to define a coherent chronometric-geometric language in which they can be asked without first assuming additional timelike dimensions.

8 Conclusion

We have proposed a phenomenological closure of a three-step program in cyclic cosmology. The first step redefined cosmic age through a light-chronometric quantity, the second step introduced inter-cycle drift, and the present step supplied the geometric interpretation. The central thesis is simple: standard 3+1 spacetime is retained, but no additional timelike dimensions are introduced. All extra freedom is treated as spatial, while cosmological evolution is encoded by monotonic chronometric order.

In this framework, a cyclic universe is not an infinite timelike loop but an infinite sequence of cycles. The distinction matters. It allows the framework to remain compatible with standard relativistic spacetime while giving a precise operational role to extra spatial freedom. It also clarifies the meaning of the previous effective speed c_{eff}: it is a chronometric quantity for the observable sector, not a claim about a changing fundamental vacuum constant.

The universe may become geometrically larger while becoming causally narrower. In that sense, the cycle is not a simple repetition but an infinite sequence of phase-shifted recurrences.

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