Operational Freeze-Out of Causal Propagation in an Effective Black-Hole Core: A Toy Model Beyond Classical Geometric Validity

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Abstract

We propose a toy model for the operational freeze-out of causal propagation in an effective black-hole core. The paper does not claim a literal microscopic description of the black-hole interior; instead, it assumes that classical geometric validity breaks down near a core region and replaces the unknown interior by an effective metric sector parameterized by an order variable ξ. For radial null propagation in

$$ds^2 = -A(r,\xi)c_0^2 dt^2 + B(r,\xi)dr^2 + r^2 d\Omega^2,$$

we define an effective causal speed by

$$c_{\text{eff}}(r,\xi) = c_0\sqrt{\frac{A(r,\xi)}{B(r,\xi)}}.$$

A near-core power-law ansatz then yields a simple freeze-out criterion: when the ratio A/B vanishes fast enough, the communication time from the core to any finite shell diverges, even though the fundamental vacuum constant c_0 remains fixed. This provides an operational sense in which light or information can become effectively immobile without invoking a literal variation of the vacuum light speed. An optional thermodynamic interpretation is discussed in terms of an effective bounded subsystem with inverse temperature $\beta_{\text{eff}} = \partial S_{\text{eff}}/\partial E$; negative-$\beta_{\text{eff}}$ sectors, if considered, are treated only as auxiliary interpretations and not as the core result. The model yields a compact framework for discussing remnant-like behavior, late-time information immobilization, and possible modifications of near-core causal structure while remaining agnostic about the true microscopic completion.

1 Introduction

Black holes are simultaneously among the most tightly constrained and the least directly accessible objects in theoretical physics. The exterior thermodynamics of stationary black holes is well established: Hawking radiation assigns a temperature to the horizon, and the Hawking temperature decreases as the black-hole mass increases [2, 1]. For astrophysical black holes, the horizon temperature is therefore extremely low; the high temperatures usually associated with observed black-hole systems belong instead to accretion flows and jets outside the horizon.

At the same time, the classical black-hole interior is not a safe arena for literal extrapolation. The appearance of singular behavior in general relativity is commonly interpreted as a sign that the classical geometric description has reached the edge of its validity. This motivates a conservative question: not "what is the true interior?" but rather "what effective interior structure would be sufficient to make causal propagation operationally freeze out near a core region?"

We do not claim a literal description of the black-hole singular interior. We only ask what effective interior geometry would be sufficient to produce operational freeze-out of causal propagation near a core region.

This viewpoint shifts the problem from metaphysical speculation to phenomenological classification. Instead of asserting that light literally ceases to exist or that the vacuum light speed changes, we ask under what conditions an effective interior medium or geometry makes communication from a core neighborhood take arbitrarily long when measured by the exterior time coordinate. In this sense, our notion of "stopping light" is operational rather than ontological.

The paper is organized as follows. Section 2 separates standard input from speculative assumptions. Section 3 defines the effective interior metric and the associated effective causal speed. Section 4 derives the freeze-out criterion and the divergence of the core-to-shell crossing time. Section 5 discusses an optional thermodynamic interpretation in terms of a bounded effective subsystem. Section 6 outlines possible theoretical consequences, while Section 7 states clearly what the model does and does not claim.

2 Standard input versus speculative input

The model rests on a deliberately asymmetric foundation. Some ingredients are standard and widely accepted:

(i) black-hole horizon thermodynamics, in particular the inverse proportionality of the Hawking temperature to black-hole mass [2, 1];

(ii) the fact that singular behavior in classical relativity signals the breakdown of the classical description rather than a completed microscopic theory [3, 4];

(iii) the existence of effective-medium systems in which signal propagation can be dramatically slowed without altering the fundamental vacuum constant c_0, as in slow-light and stopped-light experiments [5, 6].

The speculative input is narrower:

(i) there exists an *effective* near-core geometry characterized by an order variable ξ;

(ii) radial causal propagation in that effective region can be encoded by a sector-dependent speed $c_{\text{eff}}(r, \xi)$;

(iii) in some effective bounded subsystem, a negative inverse temperature $\beta_{\text{eff}} < 0$ may serve as an optional interpretive label, but it is not required for the main freeze-out result.

The paper therefore does not attempt to solve quantum gravity. It only constructs a minimal language in which operational causal freeze-out can be discussed cleanly.

3 Effective interior model

We assume a static, spherically symmetric effective line element in the form

$$ds^2 = -A(r,\xi)c_0^2 dt^2 + B(r,\xi)dr^2 + r^2 d\Omega^2, \tag{1}$$

where $A(r,\xi) > 0$ and $B(r,\xi) > 0$ in the region of interest, and where ξ denotes an order parameter summarizing the effective state of the interior. Depending on the microscopic completion, ξ could encode a condensate-like phase, a quantum-geometric order parameter, or the effective participation of extra spatial freedom. At the present level, it is simply a control variable.

For radial null propagation, $ds^2 = 0$ and $d\Omega = 0$, so Eq. (1) gives

$$\frac{dr}{dt} = c_0 \sqrt{\frac{A(r,\xi)}{B(r,\xi)}}. \tag{2}$$

This motivates the effective causal speed

$$c_{\text{eff}}(r,\xi) \equiv c_0 \sqrt{\frac{A(r,\xi)}{B(r,\xi)}}. \tag{3}$$

Equation (3) is the natural interior analogue of the effective propagation speeds introduced in broader chronometric phenomenology: the vacuum constant c_0 remains fixed, while the observable propagation speed becomes sector- and state-dependent.

A simple near-core ansatz is

$$\frac{A(r,\xi)}{B(r,\xi)} = \alpha(\xi) \left(\frac{r - r_c}{L}\right)^{2\lambda}, \qquad \alpha(\xi) > 0, \tag{4}$$

where r_c is the effective core radius, L is a characteristic interior length scale, and λ controls how rapidly the causal cone narrows near the core. Combining Eqs. (3) and (4) yields

$$c_{\text{eff}}(r,\xi) = c_0 \sqrt{\alpha(\xi)} \left(\frac{r - r_c}{L}\right)^{\lambda}. \tag{5}$$

The qualitative effect of different values of λ is shown in Fig. 1.

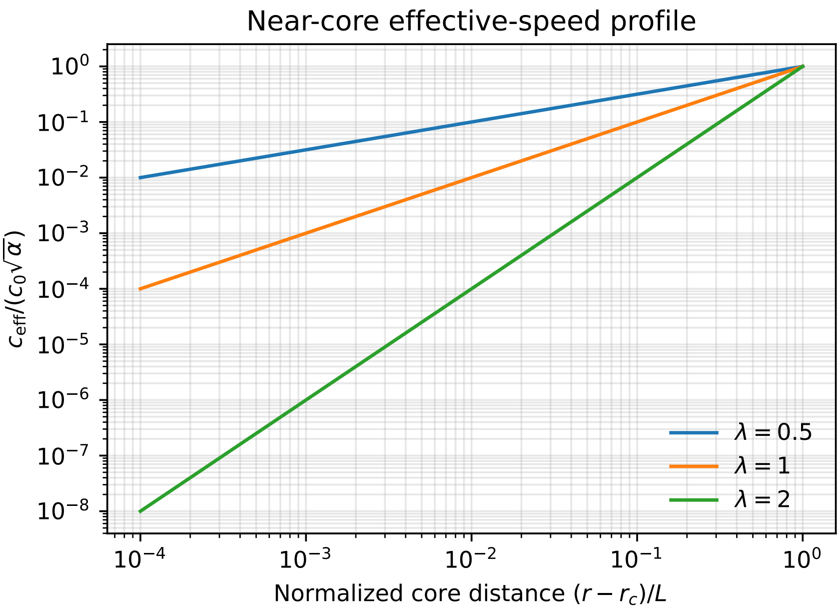

Figure 1: Near-core effective-speed profile from Eq. (5), shown in normalized units for representative exponents λ. Larger λ suppresses causal propagation more strongly as the core radius is approached.

4 Freeze-out condition

The operational question is not whether a null ray formally exists, but whether information emitted from an arbitrarily small neighborhood of the core can reach a finite radius within finite coordinate time. Let $r_1 > r_c$ be a finite shell and let $\delta > 0$ be the inner cutoff, so that the signal starts from $r = r_c + \delta$. The core-to-shell crossing time is

$$t_{\mathrm{cross}}(\delta) = \int_{r_c+\delta}^{r_1} \frac{dr}{c_{\mathrm{eff}}(r,\xi)}. \tag{6}$$

Substituting Eq. (5) gives

$$t_{\mathrm{cross}}(\delta) = \frac{L}{c_0\sqrt{\alpha(\xi)}} \int_{\delta/L}^{(r_1-r_c)/L} u^{-\lambda}\, du. \tag{7}$$

This integral can be evaluated explicitly:

$$t_{\mathrm{cross}}(\delta) = \begin{cases} \dfrac{L}{c_0\sqrt{\alpha}} \dfrac{\left(\dfrac{r_1 - r_c}{L}\right)^{1-\lambda} - \left(\dfrac{\delta}{L}\right)^{1-\lambda}}{1-\lambda}, & \lambda \neq 1, \\ \dfrac{L}{c_0\sqrt{\alpha}} \ln\left(\dfrac{r_1 - r_c}{\delta}\right), & \lambda = 1. \end{cases} \tag{8}$$

The result immediately implies the freeze-out criterion:

$$\boxed{\lambda \geq 1 \quad \Longrightarrow \quad \lim_{\delta\to 0} t_{\mathrm{cross}}(\delta) = \infty.} \tag{9}$$

When $\lambda < 1$, the crossing time remains finite; when $\lambda \geq 1$, communication from the core neighborhood to any finite shell is operationally frozen out. Figure 2 illustrates the divergence.

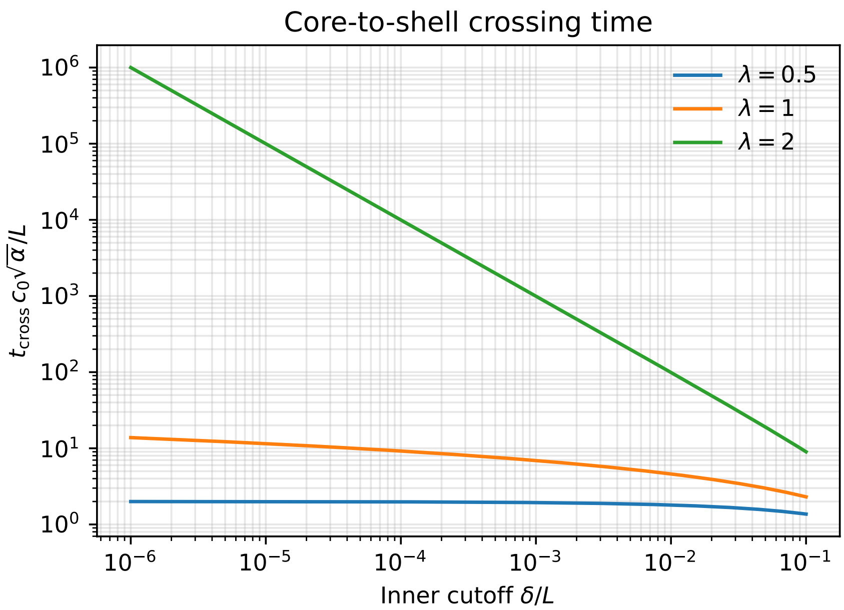

Figure 2: Dimensionless crossing time $t_{\rm cross} c_0 \sqrt{\alpha}/L$ as a function of the inner cutoff δ/L. The communication time diverges logarithmically for $\lambda = 1$ and as a power law for $\lambda > 1$.

For clarity, the same condition is summarized as a phase diagram in Fig. 3. The phrase "light stops" should therefore be read operationally: the propagation time diverges so strongly near the core that no finite-time communication protocol can extract information from arbitrarily close to the core.

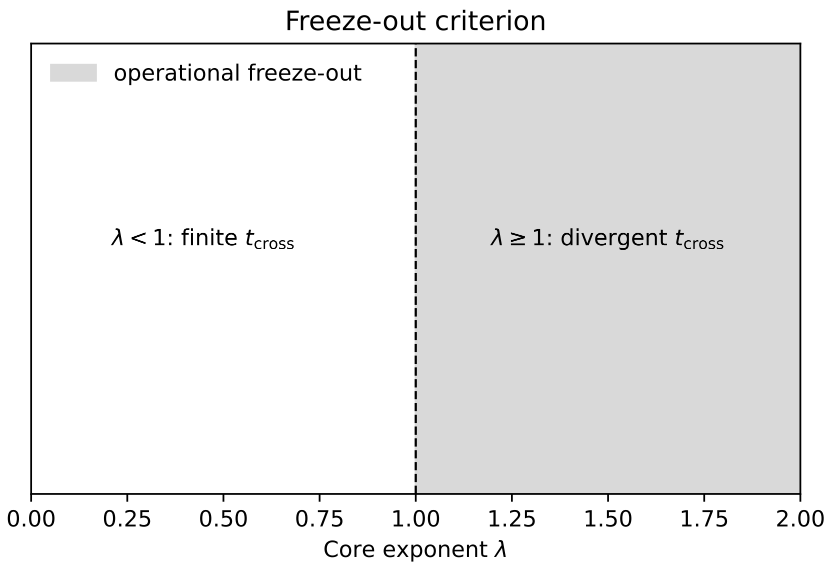

Figure 3: Freeze-out criterion from Eq. (9). The shaded region corresponds to divergent core-to-shell communication time.

5 Thermodynamic interpretation (optional)

The freeze-out result above is purely kinematic and geometric; it does not require any special thermodynamic language. Nevertheless, one may ask whether an effective thermodynamic description of the interior can provide a useful interpretation.

If the relevant interior degrees of freedom form a bounded effective subsystem, then one may define an inverse temperature by

$$\beta_{\rm eff}(E,\xi) \equiv \frac{\partial S_{\rm eff}}{\partial E}. \tag{10}$$

In such systems it is, in principle, possible to have $\beta_{\rm eff} < 0$, corresponding to an inverted population sector [7, 8]. It is crucial to state what this does *not* mean: negative temperature does not mean "colder than zero" in standard thermodynamics. It means that the effective subsystem lies on the opposite side of the entropy–energy slope, and is therefore formally hotter than any positive-temperature state.

For this reason, the language of negative temperature should be regarded here as optional and secondary. The main result of this paper is the freeze-out criterion (9). If a bounded effective subsystem exists, then a negative-$\beta_{\rm eff}$ interpretation may serve as a suggestive thermodynamic companion, but the kinematic conclusion does not depend on it.

6 Possible theoretical consequences

Although the model is not tied to one specific ultraviolet completion, it points toward a few qualitative consequences worth recording.

First, a freeze-out region could act as an *information-immobilization layer*. This is stronger than ordinary horizon trapping in the sense that even local propagation from the core neighborhood becomes operationally inaccessible. Such behavior could support remnant-like scenarios or very long-lived interior storage phases.

Second, if the freeze-out region couples imperfectly to the exterior geometry, one could imagine delayed or filtered communication between the core and the near-horizon region. In more detailed models this might modify late-time ringdown or produce echo-like phenomena. The present paper does not derive such effects, but it identifies the geometric condition under which they become plausible.

Third, if evaporation drives the system toward a regime in which λ increases, the end stage of evaporation need not be described as a simple runaway temperature increase. Instead, the effective causal cone could collapse faster than energy escapes, producing a quasi-static late phase. Again, this is not claimed as a prediction of the toy model; it is a direction suggested by the freeze-out criterion.

7 Discussion

The model is intentionally narrow. It does not claim that Eq. (1) is the true interior metric of a black hole. It does not derive $\alpha(\xi)$ or $\lambda(\xi)$ from a microscopic theory. It does not solve the information paradox. And it does not promote negative temperature to a universal feature of black holes. Its role is simpler: it provides a minimal, internally consistent vocabulary in which one can say what "operationally stopped light" means.

This precision is useful because several intuitions that are often run together are in fact different. "Light cannot escape" is not the same as "light cannot move." "The vacuum constant c_0 is fixed" is not the same as "the effective propagation speed must remain equal to c_0 in all sectors." "A singularity appears in classical GR" is not the same as "the classical metric must remain valid all the way to the singular point." By separating these statements, the toy model turns a diffuse intuition into a calculable criterion.

The model also fits naturally with a broader chronometric perspective in which one retains the standard relativistic vacuum constant while allowing state-dependent effective propagation in specific sectors. In that sense, the present paper may be read as a local interior analogue of earlier chronometric constructions, now applied to a near-core geometry rather than to cosmological evolution.

8 Conclusion

We have formulated a toy model for operational freeze-out of causal propagation in an effective black-hole core. Starting from a static spherically symmetric effective metric, we defined a sector-dependent causal speed

$$c_{\text{eff}}(r, \xi) = c_0 \sqrt{A/B},$$

and showed that a simple near-core power-law ansatz leads to a sharp criterion: if the narrowing exponent satisfies $\lambda \geq 1$, then the communication time from an arbitrarily small core neighborhood to a finite shell diverges. In this operational sense, light and information become effectively immobile near the core, even though the vacuum constant c_0 is not altered.

The paper therefore does not claim a microscopic theory of the black-hole interior. It provides instead a compact phenomenological answer to a specific question: what effective geometry would be sufficient to freeze out causal propagation? That question can be posed, and answered, without literal singular-core realism, without modifying the vacuum light constant, and without relying on negative-temperature language as a central premise. Those who wish may add bounded-subsystem thermodynamics as an auxiliary interpretation, but the freeze-out criterion already stands on its own.

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