Multiverse Balance Laws in Cyclic Cosmology: An Open-Subsystem Formulation for Energy and Entropy Exchange

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May 12, 2026

Abstract

We formulate a balance-law framework for cyclic cosmology in which any individual universe U_i is treated as an open subsystem of a larger multiverse ensemble. The goal is not to prove a microscopic mechanism for inter-universe transfer, but to provide a mathematically consistent language in which local apparent non-conservation can coexist with global closure. Standard $3+1$ spacetime is retained within each universe, no additional timelike dimensions are introduced, and evolution is ordered by a monotonic chronometric parameter rather than by a timelike loop. For an ensemble $\mathcal{U} = \{U_i\}$, each member is assigned a coarse-grained state vector containing an effective energy E_i, entropy S_i, baryon asymmetry $\eta_{B,i}$, memory parameter ε_i, and an extra-spatial-freedom variable χ_i. The central proposal is an antisymmetric transfer law

$$\frac{\mathrm{d}E_i}{\mathrm{d}\lambda} = \sum_{j\neq i} J_{ji}(\lambda), \qquad J_{ij} = -J_{ji},$$

which implies exact ensemble closure $\sum_i \mathrm{d}E_i/\mathrm{d}\lambda = 0$ under mild convergence assumptions. Entropy is treated separately by

$$\frac{\mathrm{d}S_i}{\mathrm{d}\lambda} = \Pi_i(\lambda) + \sum_{j\neq i} \Gamma_{ji}(\lambda), \qquad \Pi_i \geq 0, \qquad \Gamma_{ij} = -\Gamma_{ji},$$

so that the ensemble can be closed in energy while remaining thermodynamically nontrivial. This paper supplies the balance-law layer beneath earlier light-chronometric, inter-cycle-drift, and chronometric-geometric constructions, and prepares the ground for a later seed-state functional and transfer-channel analysis.

1 Introduction

Cyclic cosmology is not a single theory but a family of scenarios in which expansion, contraction, bounce, or conformal crossover recur in some effective sense. Existing models already show that the words "same cycle" and "next cycle" can mean very different things depending on which quantity is followed. In some constructions the maximal scale factor grows, in others local bang-scale quantities recur while horizon structure resets, and in still others the physically relevant ordering is better understood as a succession of phase-shifted recurrences than as a literal closed timelike loop [1, 2, 3].

The three companion manuscripts preceding the present one were written in that spirit. The first redefined cosmic age by a light-chronometric quantity,

$$\tau(t) = \int_0^t c_{\mathrm{eff}}(t')\,\mathrm{d}t', \tag{1}$$

where c_{eff} is a sector-dependent effective causal speed rather than a literal variation of the vacuum constant c_0. The second introduced inter-cycle drift through a minimal cycle genome $(\chi_n, \eta_{B,n}, \varepsilon_n)$ and showed that geometric size and causal size can drift in opposite directions from cycle to cycle. The third clarified the geometric meaning of that construction: standard $3+1$ spacetime is retained, all extra degrees of freedom are treated as spatial, and cosmological evolution is encoded by a monotonic chronometric order rather than by reversible motion along an added time axis [4, 5, 6].

What remains missing is a balance-law layer. The previous papers describe how a single cycle terminates, how one cycle differs from the next, and how higher-dimensionality should be interpreted operationally. But they do not yet answer a basic bookkeeping question: if one universe is treated only as a part of a larger structure, what is the mathematically consistent relation between local drift and global closure? Put differently, if a single universe behaves as an open subsystem, how should one write the conservation law for the full multiverse ensemble?

This paper addresses that question at the level of phenomenology. It does not propose a microscopic theory of how universes exchange energy or information. It does not derive a specific bounce, tunneling, or braneworld mechanism. Instead, it introduces the smallest set of assumptions needed to write a balance law for a multiverse viewed as a closed ensemble of cyclic subsystems. The main idea is straightforward:

> *an individual universe is treated as an open subsystem, while the multiverse ensemble is treated as the closed system in which the net transfer balances exactly.*

The resulting framework is intentionally conservative in three respects. First, it retains the standard relativistic description of each universe in $3+1$ spacetime. Second, it introduces no additional timelike dimensions. Third, it separates the exact closure of ensemble energy from the nontrivial behavior of entropy, allowing the global system to be closed in energy while still producing irreversibility and drift at the subsystem level.

The purpose of the paper is therefore twofold. First, to formulate a minimal balance-law language for multiverse cyclicity. Second, to connect that language to the state variables already introduced in the earlier companion papers. The result is not a final theory of multiverse dynamics. It is a framework in which later seed-state and transfer-channel models can be embedded without first confusing local open-system behavior with global closure.

2 Open universes, closed multiverse: the balance-law framework

2.1 Ensemble, ordering parameter, and state variables

We consider an ensemble of universes

$$\mathcal{U} = \{U_i\}_{i\in I}, \tag{2}$$

where I may be finite or countably infinite. Each U_i is treated as a coarse-grained subsystem possessing at least the following state variables:

$$X_i(\lambda) = \big(E_i(\lambda),\, S_i(\lambda),\, \chi_i(\lambda),\, \eta_{B,i}(\lambda),\, \varepsilon_i(\lambda)\big). \tag{3}$$

Here E_i denotes an effective energy assigned to the subsystem, S_i its coarse-grained entropy, χ_i the effective participation of extra spatial freedom, $\eta_{B,i}$ the baryon asymmetry, and ε_i a memory or copying-noise parameter.

A crucial point is that the ensemble is not ordered by a closed timelike loop. Instead, we introduce a monotonic ordering parameter λ for the multiverse ensemble and require that each local chronometric variable τ_i satisfy

$$\frac{\mathrm{d}\tau_i}{\mathrm{d}\lambda} > 0. \tag{4}$$

Equation (4) expresses the same principle adopted in the chronometric-geometric paper: evolution is future-oriented and ordered, but no additional timelike axis is introduced. The multiverse therefore contains an infinite sequence of cycles, not an infinite timelike loop.

The present formulation deliberately does not assume that all universes share the same local metric, matter content, or cycle timescale. The only common structure is the existence of the ordering parameter λ and the possibility of effective transfer between subsystems. This is enough to formulate a consistent balance law.

2.2 Exact energy closure for the ensemble

The simplest exact closure is obtained by introducing pairwise transfer rates $J_{ij}(\lambda)$ with the antisymmetry condition

$$J_{ij}(\lambda) = -J_{ji}(\lambda). \tag{5}$$

The effective energy of universe U_i then satisfies

$$\frac{\mathrm{d}E_i}{\mathrm{d}\lambda} = \sum_{j\neq i} J_{ji}(\lambda). \tag{6}$$

Equation (6) means that E_i changes only through exchange with the rest of the ensemble.

If the ensemble is finite, summing (6) over all i immediately gives

$$\sum_i \frac{\mathrm{d}E_i}{\mathrm{d}\lambda} = 0. \tag{7}$$

For a countably infinite ensemble, the same result holds provided the transfer series converges absolutely, for example if

$$\sum_{i<j} |J_{ij}(\lambda)| < \infty \tag{8}$$

for each λ of interest.

Proposition 1. *Assume (5) and absolute convergence (8). Then the total ensemble energy*

$$E_{\mathrm{tot}}(\lambda) = \sum_i E_i(\lambda) \tag{9}$$

is independent of λ.

Proof. Summing (6) over i and using antisymmetry gives pairwise cancellation,

$$\sum_i \sum_{j\neq i} J_{ji} = \sum_{i<j} (J_{ji} + J_{ij}) = 0,$$

with rearrangement justified by (8). Hence $\mathrm{d}E_{\mathrm{tot}}/\mathrm{d}\lambda = 0$. □

This is the basic balance law. It captures the intuition that what looks like gain or loss from the viewpoint of a single universe may simply be transfer within a larger closed ensemble.

2.3 Entropy balance for open cyclic subsystems

Energy closure does not imply thermodynamic triviality. To allow irreversible evolution inside each universe, we write the entropy balance in the form

$$\frac{\mathrm{d}S_i}{\mathrm{d}\lambda} = \Pi_i(\lambda) + \sum_{j\neq i} \Gamma_{ji}(\lambda), \qquad \Pi_i \geq 0, \qquad \Gamma_{ij} = -\Gamma_{ji}. \tag{10}$$

The term Π_i is the internal entropy-production rate of subsystem U_i, while Γ_{ij} is an entropy-exchange current between universes.

Summing (10) gives

$$\frac{\mathrm{d}S_{\mathrm{tot}}}{\mathrm{d}\lambda} = \sum_i \Pi_i(\lambda) \geq 0, \tag{11}$$

again assuming convergence of the exchange series. Thus the multiverse can be exactly closed in energy while still allowing nontrivial entropy production. This is precisely the structure one expects from a closed ensemble made of open nonequilibrium subsystems.

The distinction between (7) and (11) is conceptually important. It prevents the common confusion that any attempt to close the multiverse in energy must also erase irreversibility. It does not. Exact energy closure and positive entropy production can coexist.

2.4 Embedding the cycle genome

The cycle-genome variables introduced previously now acquire a natural interpretation within the balance-law setting. The baryon asymmetry $\eta_{B,i}$ and memory variable ε_i are subsystem variables that may drift because the universe is open at the cycle level, even though the ensemble is globally closed. The variable χ_i summarizes how strongly extra spatial freedom participates in the effective reconstruction of the next cycle.

At this stage we do not specify the microscopic transformation law for $(\chi_i, \eta_{B,i}, \varepsilon_i)$ in terms of the currents J_{ij} and Γ_{ij}. That step is deferred to a later seed-state or transfer-channel paper. The present result is more basic: it establishes the mathematical setting in which such a transformation law can be written without confusing subsystem drift with violation of global closure.

Status of the present note. The balance law formulated here should be read as the foundational layer of a multiverse cyclic entropy model. It does not yet derive a specific seed functional or boundary transfer operator. Its role is to define the bookkeeping architecture: one universe is an open subsystem; the multiverse is the closed ensemble.

References

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